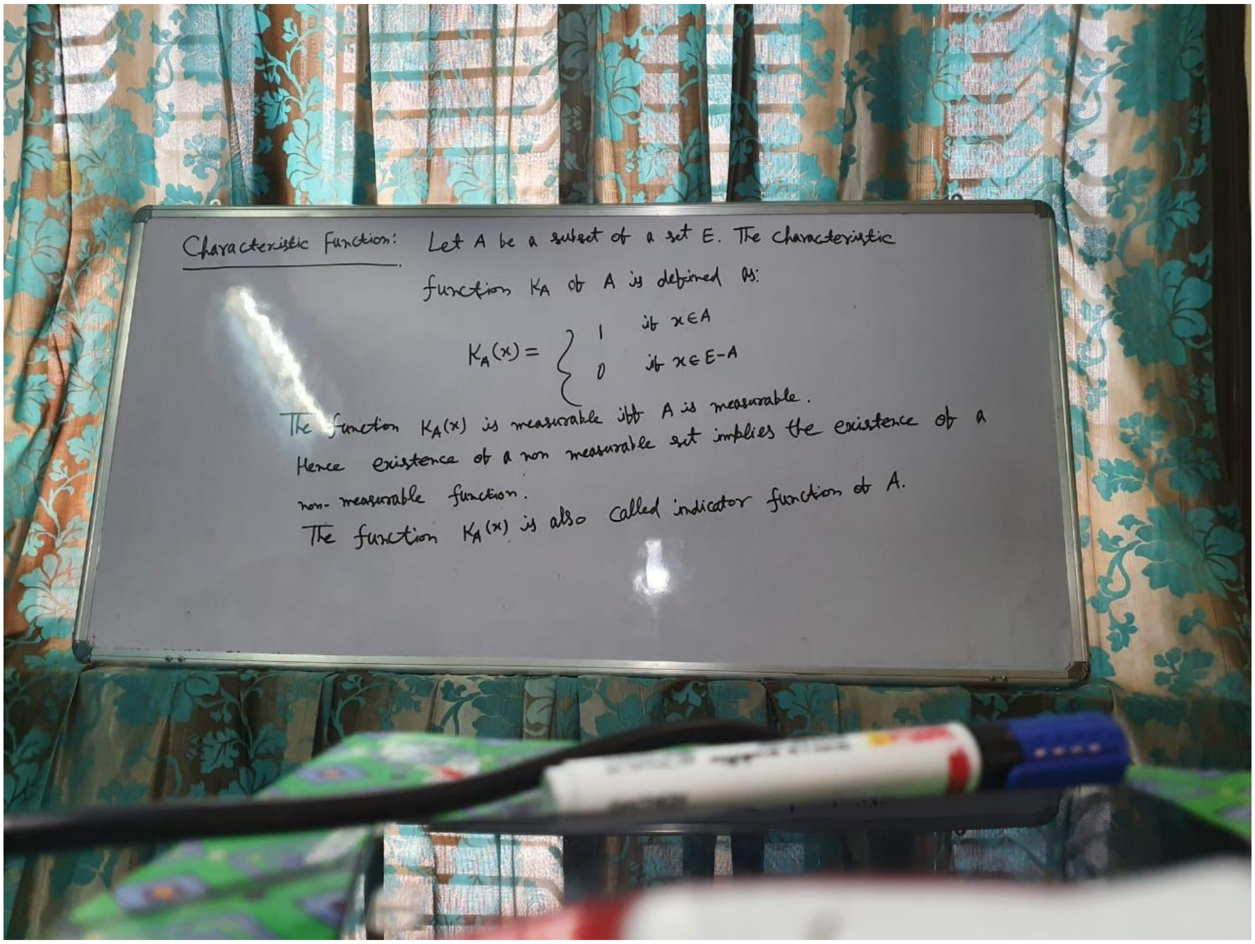

Characteristic Function: Let A be a subset of a set E. The characteristic function K_A of A is defined as:

$$K_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \in E-A \end{cases}$$

The function $K_A(x)$ is measurable iff A is measurable.

Hence existence of a non measurable set implies the existence of a non-measurable function.

The function $K_A(x)$ is also called indicator function of A.

Simple Function:- A function is said to be simple function if the range of the function is finite.

A real valued function ϕ is called simple if it is measurable and assumes only a finite number of values.

If ϕ is simple and has finite number of values $\alpha_1, \alpha_2, \cdots \alpha_n$ then ϕ is expressible as

$$\phi(x) = \sum_{i=1}^{n} a_i \chi_{A_i}$$

where $A_i = \{x : \phi(x) = \alpha_i\}$

Step Function: A real values functions defined on an interval $[a, b]$ is said to be step function if there exists a partition $a = x_0 < x_1 < \cdots < x_n = b$

s.t the function assumes one and only one value in each interval.

Thus step function also assumes finite number of values like simple functions but here the set $\{x : s(x) = c_i\}$ are intervals for each i

every step defined is also a simple function but the Converse is not true

as the $f : R \to R$ s.t. $f(x) = \begin{cases} 1, & x \text{ is rational} \\ 0, & x \text{ is irrational} \end{cases}$

is a simple function but not step as the sets of rational an irrational are not intervals.